$\mathrm{Re}^2 \frac{\chi_2}{\chi_0}) p^{(0)}$. For example, at Re = 0.1, in all cases considered in the present study $p^{(1)}$ is no more than 12% of $p^{(0)}$, while $p^{(2)}$ does not exceed 0.1% of $p^{(0)}$, i.e., in neglecting the third expansion term the error in the calculation does not exceed 0.1%.

NOTATION

R, z, φ, cylindrical coordinates of points of the mean surface of an annular channel; ξ, η, φ, dimensionless orthogonal coordinate system; ξ = const, surface of revolution corresponding to the mean surface of the channel; η, φ, meridional and angular coordinates at the mean surface, $\eta = \eta_1$ and $\eta = \eta_2$, boundaries of the annular channel; $H_\eta(\eta, \xi)$, $H_\varphi(\eta, \xi)$, dimensionless Lamé coefficients; p, pressure; ρ, fluid density; ν, kinetic viscosity coefficient, V, characteristic flow velocity; Q, total fluid discharge through the annular channel; C_0, C_n, C_{-n}, coefficients of expansion (4) in a Fourier series; $\chi_n(\eta, \xi)$, asymptotic expansion coefficients of the dimensionless pressure Π, defined by Eqs. (2), (7), and (8) for n = 0, 1, 2; and Δp, pressure drop in the channel.

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STABILITY OF THE BOUNDARY LAYER OF LIQUID UNDER A NONUNIFORM TEMPERATURE DISTRIBUTION OF THE SURFACE

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UDC 532.526

We study the effect of a longitudinal gradient of the surface temperature on the stability of the boundary layer of an incompressible liquid. A comparison shows a good agreement of the results with experimental data.

Only a relatively small number of works have been devoted to the problem of stability and the transition to the turbulent regime in a boundary layer of an incompressible liquid at a surface with heat-exchange [1-9]. It was noted in the first investigations (which were carried out for water [1, 2]) that the surface temperature affects considerably the transition process. The nature of this influence is opposite to that which is observed in gases. This is caused by the decrease of water viscosity with increasing temperature. Cooling causes the appearance of an inflection point in the mean velocity profile near the wall and, consequently, it destabilizes the flow. Heating, on the other hand, gives a fuller velocity profile and, accordingly, it stabilizes the flow.

A sufficiently strong dependence of the stability characteristics (the minimum critical Reynolds numbers and the coefficients of spatial growth of the perturbations) for a surface layer of water on the superheating of the surface gives grounds for expectations that, for an appropriate temperature distribution along the surface, a considerable increase or decrease of the flow stability can be obtained. Detailed investigations of this problem can play an important role in the solution of the control of the boundary layer. The practical importance of the problem is confirmed also by the results of the experimental work [8] which have a preliminary character and indicate that the transition of a laminar boundary layer to a turbulent one depends, to a considerable degree, on the longitudinal temperature gradient at the wall.

In the present work we study the effect of nonuniformity of the surface temperature distribution on the development of small perturbations in a laminar boundary layer of an incompressible liquid. It is shown that, when the total heat flux remains unchanged, the posi-

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tion of the point where stability is lost depends considerably on the longitudinal temperature gradient.

FORMULATION OF THE PROBLEM

We consider the stability of a planar boundary layer of a viscous incompressible liquid with a nonuniform temperature distribution. We write down the Navier–Stokes equatins, the equation of continuity and the energy equation neglecting viscous dissipation:

$$\rho\frac{Du}{Dt}=-\frac{\partial p}{\partial x}+2\frac{\partial}{\partial x}\left(\mu\frac{\partial u}{\partial x}\right)+\frac{\partial}{\partial y}\left[\mu\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)\right],$$
$$\rho\frac{Dv}{Dt}=-\frac{\partial p}{\partial y}+\frac{\partial}{\partial x}\left[\mu\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)\right]+2\frac{\partial}{\partial y}\left(\mu\frac{\partial v}{\partial y}\right),$$
$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0,\quad c_p\rho\frac{DT}{Dt}=\frac{\partial}{\partial x}\left(\lambda\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(\lambda\frac{\partial T}{\partial y}\right). \tag{1}$$

Here

$$\frac{D}{Dt}=\frac{\partial}{\partial t}+u\frac{\partial}{\partial x}+v\frac{\partial}{\partial y}.$$

The boundary conditions are

$$u=0,\quad v=0,\quad T=T_w\ (y=0),\quad u\to u_e,\quad T\to T_e\ (y\to\infty). \tag{2}$$

The surface temperature will be specified in the form

$$T_w=T_e+T_1(a+\xi^n). \tag{3}$$

Here, $\xi = x/L$. The Prandtl number and the viscosity of the liquid are known functions of the temperature

Linearizing the system of equations (1) we obtain equations which describe the development of small perturbations in the boundary layer. It is assumed that the characteristic length of the temperature change, which is commensurable with the size of the surface L, is much longer than the wavelength of the proper perturbation (the Tolmin–Schlikhting wave) λ_0. This assumption makes it possible to use the approximation of a plane-parallel boundary layer, and to represent the perturbation of the flow function in the form of a plane wave $\varphi(y)\exp(i\alpha(x-ct))$. The equations for the perturbations can be written in the form of a single relation

$$i\alpha R[(u-c)(\varphi''-\alpha^2\varphi)-u''\varphi]=\gamma(\varphi^{IV}-2\alpha^2\varphi''+\alpha^4\varphi)+2\gamma'(\varphi'''-\alpha^2\varphi')+\gamma''(\varphi''+\alpha^2\varphi). \tag{4}$$

The boundary conditions are

$$\varphi=0,\quad \varphi'=0\ (\eta=0),\quad \varphi\to 0,\quad \varphi'\to 0\ (\eta\to\infty). \tag{5}$$

Here and below, the prime indicates differentiation with respect to the dimensionless coordinate $\eta = y(u_e/x\nu_e)^{1/2}$. It is known that this transformation gives equations of the boundary layer which allow, under certain conditions, an automodelling solution.

It should also be noted that the equation for perturbations (4) contains terms with the viscosity distribution and its derivatives which, alongside the velocity profile, affect strongly the stability characteristics of the boundary layer of water [3]. When the viscosity is constant (γ = 1), relation (4) reduces to the known Orr–Sommerfeld equation.

Thus, the problem is reduced to the determination of eigenvalues of Eq. (4) with the homogeneous boundary conditions (5).

The solution of the boundary-value problem (4) and (5) requires the knowledge of the functions in the coefficients of relation (4). These are the distributions of the average velocity and viscosity over the thickness of the boundary layer, and also their first and second derivatives with respect to the transverse coordinate. These distributions can be found by solving the equations of the thermal boundary layer which, in terms of the variables ξ and η, have the form

$$\frac{\partial}{\partial\eta}(\gamma f'')'+\frac{m+1}{2}f''f+m(1-f'^2)=\Phi_1,$$
$$\frac{\partial}{\partial\eta}\left(\frac{\gamma}{\Pr}\Theta'\right)+\frac{m+1}{2}f\Theta'+n(1-\Theta)f'\frac{\xi^n}{a+\xi^n}=\Phi_2, \tag{6}$$

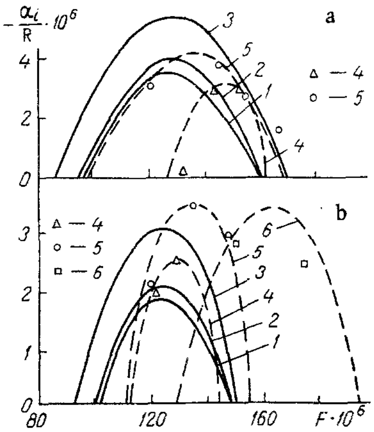

Fig. 1. Coefficients of the spatial growth of perturbations in the case of a power temperature distribution for various values of the parameter n, and R = 800. The full line corresponds to numerical calculations, and the dashed lines corresponds to an approximation of the experimental data of [8]: a) $T_w - T_e$ = 1.67°C; 1, 4) n = −0.2; 2, 5) 0; 3) 3; b) $T_w - T_e$ = 2.78°C; 1, 4) n = −0.2; 2, 5) 0; 3) 1.

$$\Phi_1 = \xi\left(f'\frac{\partial f'}{\partial\xi} - f''\frac{\partial f}{\partial\xi}\right),\quad \Phi_2 = \xi\left(f'\frac{\partial\Theta}{\partial\xi} - \Theta'\frac{\partial f}{\partial\xi}\right).$$

The boundary conditions are

$$f=0,\quad f'=0,\quad \Theta=0\ (\eta=0),\quad f'\to1,\quad \Theta\to1\ (\eta\to\infty), \tag{7}$$

where

$$\Theta=\frac{T-T_w}{T_e-T_w},\quad m=\frac{\xi}{u_e}\frac{du_e}{d\xi},\quad f=\int_0^\eta\frac{u}{u_e}\,d\eta.$$

It should be noted that for n = 0, the right-hand sides of the equations of the system (6) vanish, and the system takes the automodeling form.

An estimate of the effect of a particular temperature distribution on the stability requires a comparison of the different distributions while fixing either the total amount of the energy supplied, or the temperature in some given point of the surface. In the present work, the comparison is carried out for equal integrated thermal fluxes at the wall. It was noted above that, in this case, one can umambiguously establish the most favorable distribution from the viewpoint of an increase of stability of the boundary layer. The equality of the heat fluxes is written as follows:

$$\int_0^1\left(\lambda\frac{\partial T}{\partial y}\right)_w d\xi=\int_0^1\left(\lambda_0\frac{\partial T_0}{\partial y}\right)_w d\xi.$$

The relationship between the homogeneous and inhomogeneous temperature distribution is then obtained from

$$T_{10}=BT_1,$$

$$B=\frac{1+\beta}{(1+a_0)\Theta'_{0w}}\frac{\mathrm{Pr}_{0w}}{\mu_{0w}}\int_0^1\frac{\mu_w}{\mathrm{Pr}_w}(a+\xi^n)\,\xi^\beta\Theta'_w\,d\xi.$$

Here, $\beta = (m - 1)/2$.

DISCUSSION OF THE OBTAINED RESULTS

The stability of a boundary layer of an incompressible liquid with a surface temperature distribution of the form (3) was studied using numerical calculations on a computer. Water was chosen as the working substance. The eigenvalue problem (4) and (5) was solved by the orthogonalization method [10]. The profiles of average velocity and temperature were determined from the solution of the system of equations (6) with the boundary conditions (7), which was sought by the Keller method in the form of a grid function [11].

The results of the calculations of the stability characteristics (m = 0) were represented in the form of neutral stability curves F = F(R), where $F = \alpha c\nu_e/u_e^2$, and in the form of dependences

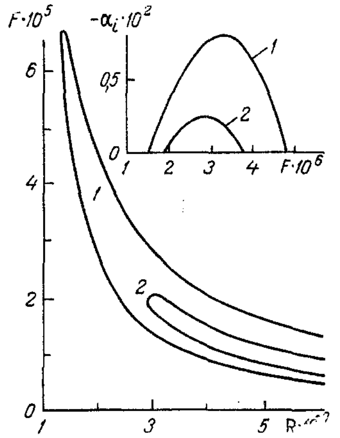

Fig. 2

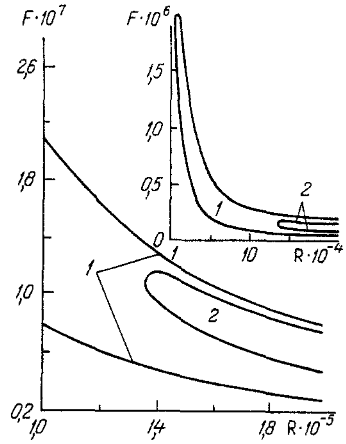

Fig. 3

Fig. 2. Curves of neutral stability and the spatial growth coefficients of the perturbations for a fixed value of the Reynolds number $R = 10^4$ when the surface is heated according to the law (3): 1) uniform temperature distribution for $a = 0$, $n = 0$, $T_1/T_e = 0.019$; 2) nonuniform distribution for $a = 0$, $n = 1$, $T_1/T_e = 0.0353$.

Fig. 3. Curves of the neutral stability in the case of strong superheating of the surface according to the law (3): 1) uniform temperature distribution for $a = 0$, $T_1/T_e = 0.097$; 2) nonuniform distribution for $a = 0$, $n = 1$, $T_1/T_e = 0.177$.

of the coefficients of spatial growth of perturbations $\alpha_i(F)$ for fixed values of the Reynolds numbers.

In the first stage of the investigation the growth coefficients α_i, determined by the numerical calculation, were compared with the experimental data obtained in [8]. The results of this comparison are shown in Fig. 1. The experimental data were obtained in a weakly turbulent water tube with the degree of turbulence of the free flow equal to 0.1-0.2%. Small perturbations were introduced into the boundary layer near the surface of a circumfluous flat plate by using a vibrating strip. The measurements of the growth coefficients were carried out for a fixed temperature of the wall in a given point $T_w(\xi) - T_e = 1.67°C$, Fig. 1a, $T_w(\xi) - T_e = 2.78°C$, Fig. 1b) and different values of the parameter n for R = 800. It is seen from Fig. 1 that there is a sufficiently good agreement of the theoretical and experimental curves which gives grounds to assume that the obtained results are reliable.

HEATING OF THE SURFACE

Figures 2 and 3 show the stability characteristics of the boundary layer of water under surface heating for different values of the parameters.

It is seen from Fig. 2 that, under a nonuniform heating of the surface, the region of unstable frequencies is narrowed considerably. The minimum critical Reynolds number R* is approximately equal to $2.8 \cdot 10^3$ while under uniform heating, $R^* = 1.2 \cdot 10^3$. This indicates that the length of the laminar segment increases by almost a factor of 6. For lower values of the parameter F, the difference is not so large, but it is still considerable. For example, for $F = 1 \cdot 10^{-5}$, the length which corresponds to the beginning of the growth of perturbations under a nonuniform heating increases by almost a factor of 1.5. The nonuniform heating increases by almost a factor of 1.5. The nonuniformity of the surface temperature distribution shows an analogous stabilizing influence on the velocity of growth of the perturbations. For a chosen Reynolds number, α_i decreases by almost a factor of 3.5.

Figure 3 corresponds to the case when the maximum superheating of the surface is equal to 50°C. It is seen that, in this case, the boundary layer becomes more sensitive to nonuniformities of the temperature distribution. The critical Reynolds number increases by almost an order of magnitude ($R^* = 1.37 \cdot 10^5$ as compared with $R^* = 1.6 \cdot 10^4$ under uniform heating)

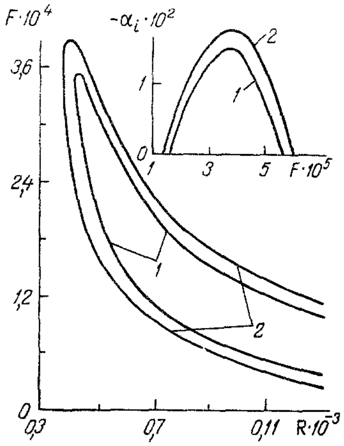

Fig. 4. Curves of neutral stability and the coefficients of spatial growth of perturbations for a fixed value of the Reynolds number $R = 2 \cdot 10^3$ when the surface is cooled according to the law (3): 1) uniform temperature distribution for $a = 0$, $n = 0$, $T_1/T_e = 0.009$; 2) nonuniform distribution for $a = 0$, $n = 1$, $T_1/T_e = 0.0177$.

which corresponds to an increase of the length of the laminar segment by a factor of 65. For lower values of the parameter F, the difference of the Reynolds numbers also remains considerable. For example, for $F = 6.0 \cdot 10^{-8}$, the length which corresponds to the beginning of the growth of perturbations increases by a factor of two under nonuniform heating.

COOLING OF THE SURFACE

As was to be expected, nonuniform cooling reduces the stability of the boundary layer. The critical Reynolds number decreases ($R^* = 370$ as compared with $R^* = 420$ under uniform cooling) and the range of unstable frequencies is reduced. The quantity α_i increases approximately by a factor of 1.2. The insignificant difference of the curves 1 and 2 in the case of cooling is explained by the fact that, for n = 1, the maximum supercooling of the surface is small and is equal to only 5°C (Fig. 4).

Thus, it was established that the form of the surface temperature distribution has a considerable effect on the stability of the boundary layer of an incompressible liquid. The strong sensitivity of the boundary layer of the water to a longitudinal temperature gradient makes it possible to control effectively the state of the flow with considerable economy, in comparison with the case of uniform temperature distribution.

NOTATION

x and y, longitudinal and transverse coordinates; ξ, η, dimensionless coordinates; u, n, velocity components; p, pressure, T, temperature; ρ, density; c_p, heat capacity; μ, viscosity; λ, thermal conductivity; Pr, Prandtl number; λ_0, wavelength; α, wave number; c, phase velocity of the perturbation; and R, Reynolds number based on the displacement thickness. The subscript w corresponds to the wall, e to the external edge of the boundary layer, and 0 corresponds to a constant surface temperature.

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